Calculus Lab Manuals

Calculus I and II is taught at Portland Community College using a lecture/lab format.The laboratory time is set aside for students to investigate the topics and practice the skills that are covered during their lecture periods. These lab manuals serves as guides for the laboratory component of these courses.

HTML and PDF

Each manual has been released with several synced versions that offer different features.The essential content of each version is the same as for all others.

	MTH 251	MTH 252
Web version Whenever there is an internet connection, and you do not prefer to have a print copy, this version is most recommended. It offers interactive elements and easier navigation than a PDF could offer.	http://spot.pcc.edu/math/clm The web version offers full walk-through solutions to supplemental problems.	http://faculty.gvsu.edu/boelkinm/Home/AC/index.html This is the full eBook for Active Calculus. Our lab uses select sections of this book. Additionally our lab has a supplement which is only available in the PDF.
For-printing PDF	http://spot.pcc.edu/math/clm/clm-print.pdf To save on printing expense, this version is mostly black-and-white, and only offers short answers to the supplemental exercises (as opposed to full solutions).	Use the color PDF and print in grayscale.
Color PDF	http://spot.pcc.edu/math/clm/clm-print-color.pdf.	http://spot.pcc.edu/math/ActiveCalculus/ActiveCalculusWithPCCSupplement.pdf

Knowls

The HTML versions makes extensive use of "knowls". A knowl is similar to a link, except that instead of transporting you to a different location or a different page, the requisite information is brought to you as hidden content that is revealed. As you explore the HTML version, try clicking on knowl links that you see.

Copying Graphs, Tables, and Math Content

In the MTH 251 manual, the graphs and other images that appear may be copied in various file formats using the HTML version. Below each image are links to .png, .eps, .svg, .pdf, and .tex files that contain the image. The .eps, .svg, and .pdf files will not lose sharpness no matter how much you zoom, but typically are large files. Some of these formats may not be recognized by applications that you use. The .png file are of fairly high resolution, but will eventually lose sharpness if you zoom in too much. The .tex files contain code that can be inserted into other .tex documents to re-create the images.

The MTH 252 manual images are currently only available as svg images from the HTML version. For now if you need one of these images, it's easiest just to take a screen shot.

In both manuals, mathematical content can be copied from the HTML version. To copy math content into MS Word, right-click or control-click over the math content, and click to Show Math As MathML Code. Copy the resulting code, and Paste *Special* into Word. In the Paste Special menu, paste it as Unformatted Text. To copy math content into LaTeX source, right-click or control-click over the math content, and click to Show Math As TeX Commands.

Tables can be copied from the HTML version and pasted into applications like MS Word. However,

- Their decorations like horizontal and vertical lines might not carry over. These can be added back.
- Mathematical content within tables will not always paste correctly without a little extra effort as described below.

Accessibility

The HTML version is intended to meet or exceed all web accessibility standards.If you encounter an accessibility issue, please report it to the editor.

- All graphs and images should have meaningful alt text that communicates what a sighted person would see, without necessarily giving away anything that is intended to be deduced from the image.
- All math content is rendered using MathJax.MathJax has a contextual menu that can be accessed in several ways, depending on what operating system and browser you are using. The most common way is to right-click or control-click on some piece of math content.
- In the MathJax contextual menu, you may set options for triggering a zoom effect on math content, and also by what factor the zoom will be.
- If you change the MathJax renderer to MathML, then a screen reader will generally have success verbalizing the math content.

Tablets and Smartphones

PreTeXt documents like this lab manual are "mobile-friendly". The display adapts to whatever screen size or window size you are using. A math teacher will always recommend that you do not study from the small screen on a phone, but if it's necessary, this manual gives you that option.

Ordering Print Copies

For MTH 251, the print center order number for the PDF is #23660.
(Older editions were #23520, #23337.)

For MTH 252, the print center order number for the PDF is #23659.

Major Changes in Content

With the MTH 251 Manual, some changes to the content have been introduced with the release of the PreTeXt version. The most notable changes are:

- The numbering scheme does not match the earlier numbering scheme.This was a necessary consequence of converting to PreTeXt.
- The related rates lab has been rewritten from scratch using the DREDS approach. Both quantity and rate variables are explicitly defined at the beginning of each problem.
- In the implicit differentiation lab, a section on logarithmic differentiation has been added.
- The printed version only contains short answers to the supplemental questions rather than complete walk-through solutions.However, complete solutions may still be found in the HTML version and the screen PDF version.
- In the continuity section of the lab manual, some of the problems related to continuity are proceeded by tables that help students organize their thoughts around appropriate function values and limit values.
- There is an appendix on units of measure that spells out the nature and abbreviation for all of the quantities units used In the text. The rate units are mentioned in the appendix as that is part of the content of the course.
- The very first activity looks very different in the first two steps. The questions require unit analysis which cannot be performed without units on the constants, so the units are there. After the first two problems there is a transition explaining why units are going away (other than for conclusion purposes) and away they go.
- A few problems here and there have either changed, disappeared, or been introduced.

Bug/Typo/Suggestion Reports

If you are PCC faculty and you find something about the lab manual that you think should be changed, whether it be a typo, a mathematical error, rewording of a sentence, restructuring of an image, or whatever...you can list it here. I (Alex) will come through here periodically and work through these. Log yourself into Spaces using the Login button at the top right if you are not already logged in. (After logging in, you might be taken to the Spaces home page. To get back here, the simplest thing to do is to just follow whatever link or navigating took you to this page in the first place.) Then find the button near the top of the page to Edit this page.If you are part time, then you may need to send me an email to add you to the list of people who can edit Math SAC pages in Spaces first. Click inside the table, and use the menu at the top to add a row.

3/30/16. Fixed in source.

Where. Specifying a section name (like "Velocity" for Lab 1 Activity 1) is much more helpful than a section number or a page number. You can refer to other things like exercises by number.	What to change. Please be specific if you can. Tell me to replace "this" with "that". Or if it is a problem with the behavior of the web version, be very clear about what the issue is, and also tell me what browser you are using, what version number, and what operating system.	Status. Leave blank and Alex will update with notes about applying these changes.
In Graphical Derivatives, exercise #16	A graph has horizontal asymptotes pointing in the wrong direction.	9/8/2015. Fixed in source code. Will make it to print for AY 2016/7.
In Graphical Derivatives, exercise #16	Please include the entire line as the asymptote.	12/7/15. I have plans to create a line style that starts off sparse on one end, and gets closer to solid on the other, where the arrow would point. I think this would be best. I just need some free time to get it there. 11/3/2016. Extending to full-width lines; leaving the arrows one-directional.
In Graphical Derivatives, exercise #11-15	The asymptotes are missing on Figure 4.2.8. This one is tough since the axes are the asymptotes, but could we at the very least get the labels for the asymptotes on this figure?	10/5/2015. Added asymptotes and labels.

In web version, in Velocity, definition of Average Velocity	If you make the browser window just narrow enough so as to still not induce any rearranging of the paragraph text, then activate the zoom on the math content in the definition, then the zoom window is much too small. (Actually this happens for all math content.)	9/8/2015. Researching with MathJax user group.
In Limits and Continuity 2.2.1 Exercises, #6	The outer parentheses were omitted from (xcos(x)). Since this fundamentally changes the problem, please wait until summer to change all of the versions.	6/7/2016. Fixed in source.
	Is it possible to make a universal change to put the table numbers and captions at the tops of the tables instead of the bottoms? I've always been told we put figure numbers at the bottom and table numbers at the top. My eyes always immediately look for table numbers at the top.	10/5/2015.This is already the case for the online version. For print, there is a hitch. It could be done for standalone tables, but there are many tables that are lined up alongside graphs or blocks of text. That structure currently requires captions at the bottom and then lines them all up. It would take serious structural changes to move captions for such tables. So for now the question is, would it be better to make the change anyway for the standalone tables? Then there would be inconsistency with the other kind of table. 11/3/2016. Leaving captions below for tables and figures in 2nd edition. Still agree that ideally tables would have captions at top: http://tex.stackexchange.com/questions/3243/why-should-a-table-caption-be-placed-above-the-table
In Non-existent limits, Figure 2.7.1, 2.7.2, 2.7.3	The asymptotes are missing labels	10/5/2015. I don't see this. In a physical print edition, the online print edition, and the web version (I used Firefox), I see the vertical asymptotes all there and labeled. Is this about horizontal asymptotes? They are left out intentionally since it seemed to me they are a distraction from the topic at hand. Should they be added? 6/7/2016. Marking this as resolved.
Example 3.3.2	In the Web version, the graph and the math overlap.	10/5/2015. Fixed, I think.
Example 3.3.2	Fixing the above issue for the web version will make the math extend into the margin in the print version. Will need to address this for next printing.	11/3/2016. Looks good now in print.
Figure 4.1.2	It would be helpful if the linear piece to the far right on the graph extended all the way to (if not a little past) 7 since the students need the slope to fully answer number 1 and number 2.	10/15/15.Changed in source to go all the way to 7. Can't upload now, but soon.
Throughout the Manual but often in Lab 4	The questions/text often say things like "f has been drawn in Figure x.x.x" or "Draw g' in Figure x.x.x" but then the caption for the figure reads "y=f(x)" or "y=g'(x)". I'm a fan of referring to functions by their names but I don't understand the logic of using function names in text referring to graphs but then using equations in the captions for those graphs.This seems to be a change from the old version of the Manual, where, often, the caption to a graph was simply "f" (not "y=f(x)") and then the caption for the graph jived with the text that referred to the graph.I'm not sure about what the most logical choice is but it seems to me that it might not be precise directions to tell a student to "draw f on Figure x.x.x".I mean, maybe they'd be correct to literally draw a big "f" in the middle of the graph!But if the directions were, "draw a graph of y=f(x)", it would be unambiguously clear.	12/7/15. I believe that now, no graph has something like "f" as its caption. Most have something like "y=f(x)". Some have no caption at all or something else special. As for wording the "Draw" instructions, if you want to settle on a style choice, then identify all locations of the issue, then I can change them. But I'll need your help in identifying those locations. For the record though, my opinion is that the function's name is its name. And when making a graph with x- and y-axes, you intrinsically then have to begin referring to "x" and "y" as well, where formerly in the surrounding text that was not necessary. So I don't see the problem. (Nor do I see a problem with rewording things.)
Antiderivatives: 4.5.1	Before Exercise #9, next to figure 4.5.11, it reads, "Answer the following question..." but these directions refer to #9-16, so it should be "question**s**" Also, I wonder if all of the plural "values" and "intervals" in #9-16 should be changed to "value(s)" and "interval(s)" to allow for just one such answer.	12/7/15. Changed to plural "questions", and added parentheses as noted.

"Continuity on an Interval", 2.11	I'm concerned about how the problems in "Continuity on an Interval" are laid out, especially the last problem (#11) in the problem-set. The "list" of criteria are organized like a paragraph of sentences, rather than a list, making it hard to see all of the criteria. (Students struggle enough with these problems; we need to make the list of items very clear, not buried in a paragraph.) In the other problems of this type (#9 and #10), the criteria is easier to read but I'd still rather see one single list, rather than two columns of lists since the columns are rather close together and students might think items listed next to each other are somehow related to each other. I'm also kinda shocked at how huge the scale and labels on the graphs are: is this on purpose?! It might be good to have large grids but not necessarily ginormous scale/labels...	12/7/15. Regarding the graphs, it wasn't the scale and labels that were large; they were normal size but the web browser was magnifying the graph to fit the width. That was easy: just wrap the image in a figure. There are now additional Figures 2.11.2, 2.11.3, and 2.11.4. Since no block level enumerated object came after this in this section, this has not affected numbering. The main practical issue is with the layout is that if it is all spelled out in one vertical column of items, then this is awkward at best for fitting with the constraints on the print edition. I've changed them to be that way, and the tall single column of conditions luckily is not causing awkward page breaking in the print edition. However there has been a side effect in the print edition that you may not like. With side by side objects, any captions have to go down below the side by side objects. Here, the column of conditions is so tall, that the right panel with the graph has lots of white space. Since the caption for the Figure is down low, it seems better to push the graph down low as well. And so you are left with awkward white space above the Figure.
Supplement, 2.14	Due to the previous comment, I was curious how other "draw a graph satisfying this criteria" problems looked so checked out the supplement to Lab 2 and #6 has an issue with the first column of criteria overlapping with the second column, and #7 has bullets - apparently for each item in the list - but the bullets (at least the first one) don't seem to line-up with anything, I think making it difficult to clearly see what the items are.	12/7/15. The bullets in #7 do line up, but you have to read the items carefully. There are (were) only 4 items with some groupings together within an item. I'll change these (#6 and #7) to be as above; they will probably have the same awkward whites pace issue in the print edition though.
Constant Factor Rule, 5.3.1	This issue only occurs in the "screen-reader version" (frankly, I don't know how I ever even noticed this...).In the exercises, #4 appears in the margin (?)	12/7/15. I don't see this in the Power Rule (5.3) or the Constant Factor rule (5.4). I'm using Firefox, looking at http://spot.pcc.edu/math/clm/accessible/section-power-rule.html. If you still see this, can you say what browser, version, and OS you are using? Also, can you confirm what section you saw this in? 6/7/2016. Figured out this is about the "screen pdf". Has something to do with how multicolumn exercisegroups are handled. That will be refactored this summer. Note: look at this again later. 11/3/2016. We're not going to continue with a screen pdf. To read the document electronically, the HTML will be required. This means an active internet connection is needed to access the CSS and JS libraries. However the issues that come with a screen pdf outweigh this consideration.
Supplement, 9.8.1	The graphs given in Figures 9.8.2 and 9.8.1 don't have any scale and (unless you already know about this stuff) there's no way to know what t is or to understand why there are short line segments between (0,0) and the points.I don't know what to suggest but I think these figures are confusing...	12/7/15. Would labeling it at 2 address the issue? Labeling at 1 will make these too cluttered given the pure pictorial point they are trying to make. There is no need to explain what t is here beyond what the surrounding text says. Steve's point is only to offer an explanation as to why the hyperbolic trig functions are called "hyperbolic". 6/7/2016. Marking this as resolved.
Tangent Lines, Figure 3.2.3	The "legend" for the graph is in an opaque box that covers the "y" label on the y-axis, thus leaving the axis unlabeled.Could the box be moved or ...something so that the y-axis is labeled.	12/7/15. I slid the window right two units.

The Derivative, Example 3.3.2	In the third step in the work to find f'(x), there's a missing set of parentheses around the entire object of the limit.	10/15/15. Changed in source. Not able to upload right now, but soon.
graphs	In order to get graphs from the Lab into my weekly graded labs, I copied the PNG available in the web version into my worksheet but when the prints came back from the Print Center the graphs were faint and not particularly reader-friendly. My first thought was that it the red color of the curves was the problem but now I don't think that's it. Instead, I think it's the nature of the grid lines and the fact that I like to print my graded labs on ivory colored paper. Maybe there's nothing that can be done in the Lab but I thought I should ask about making the grid-lines a bit darker, at least in one of the many versions of each graph available in the lab.	12/7/15. Some day we can sit down together, make several graph grid options in a single tex file, print on the ivory paper, and see what works best without having an unforeseen negative impact on the other output modes. Just let me know someday when you have time.
Figure 4.3.4	The graph of a piece of g' is incorrect.I believe the concavity is incorrect and I know the point where the hole is is incorrect.If I drew a line with slope 3 through the point on g where x=-5, I would get a secant line to g.The hole should be higher, perhaps at the point (-5,4).	10/15/15. It turns out the open point on the derivative graph is at (-5, pi), not (-5, 3). We could have a totally different graph here, but it seemed like the least damage would be caused if I just labeled the coordinates of that point, which is now the case for the online version. The equation for the original curve is y=5|sin(pi /5*x)| - 2. Experiments showed that a potential future replacement could by y=-5(|sin(pi/5*x)| - 1)^2 + 3. 11/3/2016. Leaving this as indicated above with the (-5,pi) labeled ont he second graph.
Suggested schedule	Would it be possible to change the suggested schedule for week 4 and week 5.I would like to move 4.4 Higher Order Derivatives from week 4 to week 5.	12/7/15. Moved.
Solutions to supplement 6.5.1 #17	The second and third line of the solutions are messed up with e^d/dsin(x) instead of simply e^sin(x).	12/7/15. Fixed.
Example 5.9.3	Final expression should be written over a common denominator. Also, maybe change the 3t^3 term in the original function to 3t^4 to avoid the domain issue you mentioned.	12/7/15. Common denominator. Don't want to change the problem right now mid-year. Put a note about the domain, but we can revisit in summer. 11/3/2016. Common denominator there. Leaving the note about the domain. (Added a similar note to the previous example.)
(Limits 2.1.1. Exercises)	Caption should be y = g(t) ...not g(x)	12/7/15. Fixed.
2.2.1 directions for 4-6 (2.2 Limit Laws)	...can be replaced with its value base up on one of the replacement ... "up on" should be "upon"	12/7/15. Fixed.
after table 8.1.3 (end of introduction to related rates)	Please remove the parentheses around the word about in the conclusion.	12/7/15. Fixed.
DREDS acronym definition	The word "for" should be replaced with the word "four."	12/7/15. Fixed.
In example 8.2.1	The second bullet in describing the rates phrase starts with the word "drone." The word "drone" should be replaced with the word "determine."	12/7/15. Fixed.
9.3.1 Exercises, 1-6 directions	The second occurrence of the word "following" should be followed by "the procedure outlined in Algorithm 9.3.1"	12/7/15. Fixed.
9.4.1 Exercises: problem 8	After the existing directions the following should be added: Then state the local minimum and maximum points on k. Specifically address both minimum and maximum points even if one and/or the other does not exist.	12/7/15. Fixed.
Figure 9.2.2	The vertical line should not be there.	12/7/15. Fixed.
4.2.1 Exercises	Can we add "onto Figure 4.2.1" to the directions in #1. I have a student confused about the two figures and that would help. It could say, "For each given value on x draw onto Figure 4.2.1 a nice long line segment at the corresponding point on g . . .	3/30/16. Fixed in source.

4.7.48	There is a contradiction in the instructions. First it claims that the only discontinuity is at a vertical asymptote at x=2. The last bullet however claims that the limit as x-> 4 from the left = -1 while the limit as x -> 4 from the right = 1, neither of which mesh with the second bullet point which says f(4)=3. Fix: Change the first bullet to say there are two discontinuities, one at a vert. asy. at x=2 and another a x=4. Note from Steve. Looked up the original problem. Those limits are supposed to be about f', not f. No other changes necessary.	3/30/16. Fixed in source.
Limit Laws Exercises introduction	Paragraph before problems 4-6, "Use the limit laws to establish the value... shown in exampleExample 2.2.1...." Fix the unneeded "example".	3/30/16. Fixed in source.
Limits and Continuity	I've found that there are far too many exercises in this week's lab. Other weeks we can get almost everything done, but week 2 is just jam packed even when I slim the list down substantially. I'm not suggesting a correction, but more asking if others have found this to be also especially true of this lab chapter. Perhaps additional questions could simply be moved to the supplement. **yes yes yes yes!! please! YES! That lab is WAY too long!! We could make a group project out of this but I vote to empower (and *direct!*) the author and his typesetter to select ~1/3 of the problems and move them to the supplement (or just delete them if there isn't interest in creating their solutions).**	6/7/2016. Notified Steve to think about this. 11/3/2016. No change for edition 2.
Exercises on Antiderivatives (4.5.1)	The wording on the problem with Jasmine and her lab assistant is difficult for students. Introducing that sine is a periodic function implies to almost all of them that the graph of sine will explain the answer to the problem. Also, the part about Jasmine being half right is too vague for students. Perhaps instead, something like... "Recall that a periodic function is one whose y-values repeat regularly. a.) Are derivatives of any periodic function always periodic? b.) Are antiderivatives of any periodic function always periodic? c.) Only one of the answers to a or b should be "no". Draw an example of a function that would demonstrate how this "no" would occur." I'm sure you can make it more clear, but I think less is more for this question.	6/7/2016. Notified Steve to think about this. 6/11/2016. Added some clarifying wording provided by Steve.
Generally about the graphs	I think the scales on the graphs are often difficult to "see". In previous discussions about the formatting of the Lab, I've commented about how I think the scale on the axes should be in a smaller font since I think the numbers, especially the negative numbers, span so much space that it's not obvious at a casual glance which spot on the axes the numbers refer to. Now I've discovered a possible explanation about why I thought the scale on the graphs was problematic: there aren't "tics" on the axes to represent the locations that the numbers represent. Scale values like "-2" span almost two full "squares" in the grid so it's (a little bit) ambiguous exactly what spot on the axis the scale-value is representing. I think there should be "tics" on the axes so that the numbers on the scale are describing the tics.	
Intro to the Chain Rule section	In the sin(x^2) example, the variable is labeled wrong in the last sentence, "The factor of d/dz(x^2) is called a *chain rule factor*." It should say d/dx(x^2).	6/7/2016. Fixed in source.
Exercises 9.4.1 #7	The derivative should be stated "k' (x)"	6/7/2016. Fixed in source.
2.4 Limits at Infinity	In the second paragraph, the second limit should say x \to -\infty	11/3/2016. Fixed.
3.4 Derivative Units	"Akbar was given..." I would suggest changing it to $V'(2)$ instead of $V'(20)$, that way students are less likely to be able to argue that the tub should be empty after 20 minutes. The purpose is that they should be recognizing that the sign on the derivative is wrong.	11/4/2016. Changed it to 2.

8.1 Related Rates	"When the radius of the snowball is 1.4 cm and the snowball is melting at a rate of...." It should be 0.3cm^3/min, not 0.3cm^3/L	Note to Alex: This got fixed in the pdf version very early on and it is correct in what was sent to the print center (phew!). I did note that it is still errant in the interactive version as well as the pdf "viewable" version. 12/5/2016. Fixed in the 201701 print edition, and will be fixed in the corresponding HTML. (There will no longer be a screen-formatted pdf; screen users should use the HTML version.)
9.5.1 Exercises, #2-6 intro (Inflection points exercises)	Sorry if this is already here, but in the printed version I have today, I just noticed that after giving the y(x), y'(x), and y''(x) functions, we're giving g(t) and g'(t). As far as I can tell, g and g' are not relevant here, but were for 9.4.1 Exercises #2. I think this is a cut-and-paste error, but could be missing something. g(t) is the function from 9.4.1 Exercise 2-6 (Sign Tables for the First Derivative). It is definitely an error.	12/5/2016. Fixed in source, and will be gone from the new HTML to be posted after 201604 ends. Unfortunately this made it into the print copies for 201701 and forward. Also there was a formatting change which makes the extra sentence more prominent.
In Section 5 Supplement, Table 5.12.5	Table lists g(x) then f' then f''...I think it should be g' and g''	6/15/2017. Fixed.
Page 83 (in current print version has no page number)		6/15/2017. The page number is there. It's just that the section title is so long that it runs into the page number.
9.4.1 #3	k should be g	6/15/2017. Fixed.
Supplemental Homework Questions for Section 5, Derivative formulas	in current 5.12.1 Supplemental exercises for Derivative formulas #38 in the questions says s(x)=xg(x)g(x), find s'(2). However, in the solutions it finds s'(2) for s(x)=xf(x)g(x). They should probably match.	6/15/2017. Fixed.
Supplement for implicit Differentiation, 7.4	Problem #2, the fractions in the sqrt aren't rendering properly... need more brackets.	6/15/2017. The CDN for MathJax had to be discontinued in April 2017, which led to some changes we couldn't deal with mid year. Formerly, \sfrac was supported (slanted fraction) but the MathJax we are using right now doesn't. However at the next build of the HTML, we will get \sfrac back.
4.2.1: Graphical Derivatives	Minor confusion for #11-15 since they refer to functions *f* and *f'* but there are three different graphs on the same page, and all are called *f* or *f'*. #11-14 don't include a reference the figure numbers: that might help clarify.	6/15/2017. The first two plot are part of question 10. The third plot and the empty grid are for 11–15. There is a sentence introducing 11–15 that separates the plots. But to avoid confusion I will change "f" to "h" in 11–15.
4.5.1: Antiderivatives	Minor grammatical error in the "preamble" to #1 & #2 (text next to Figure 4.5.4): either "have" should be "has" since the subject is "each" (not "functions"), or the subject should be changed to a plural noun, e.g., "All of the linear functions in Figure 4.5.4 have..."	6/15/2017. Changed the "have" to "has".
7.1 Implicit Differentiation	Right before Example 7.1.2, the sentence ending in "...shown in Example 7.1.2" needs a period.	6/15/2017. Fixed.
7.1.1 General Implicit Differentiation, #7	In the text next to Figure 7.1.3, there's a missing parenthesis at the end, after "... Exercise 5."	6/15/2017. Fixed.
9.5.1 exercises, #3	I think the question should ask about the critical numbers of y', not y.	